Zaawansowane aspekty praktycznego zastosowania modeli mieszanych

1. Wykład wstępny
2. Algebra macierzowa
3. Best Linear Unbiased Prediction (BLUP): model jednocechowy z pojedynczym efektem losowym

4. Best Linear Unbiased Prediction (BLUP): model wielocechowy

5. Metody redukcji wymiarów modeli wielocechowych BLUP
6. Analiza danych powtarzalnych w czasie
7. Predykcja genomowej wartości hodowlanej
8. Model jednostopniowy
9. Estymacja parametrów modelu
10. - 15. Prezentacje zaliczeniowe

1. Wielocechowy model BLUP
2. Identyczne macierze wystąpień
3. Brakujące rekordy danych
4. Różne macierze wystąpień
5. MACE

Copyright ©2020, Joanna Szyda

Wielocechowy model BLUP

Wielocechowy model BLUP

- selekcja osobników → na podstawie wielu cech

- mBLUP (**multivariate B**est **L**inear **U**nbiased **P**rediction)
  → Henderson and Quaas (1976)
  → najlepsza metoda jednoczesnej predykcji wartości hodowlanej dla kilku cech
  → wykorzystuje informację o korelacji pomiędzy cechami
  → korelacje genetyczne
  → korelacje efektów błędu

R.A. Mrode 2005 Linear Models for the Prediction of Animal Breeding Values

Copyright ©2020, Joanna Szyda

Wielocechowy model BLUP

- Cecha 1

$$\boldsymbol{y_1} = \boldsymbol{X_1 b_1} + \boldsymbol{Z_1 a_1} + \boldsymbol{e_1}$$

- Cecha 2

$$\boldsymbol{y_2} = \boldsymbol{X_2 b_2} + \boldsymbol{Z_2 a_2} + \boldsymbol{e_2}$$

- Bi-variate model

$$\begin{bmatrix} \boldsymbol{y_1} \\ \boldsymbol{y_2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{X_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{X_2} \end{bmatrix} \begin{bmatrix} \boldsymbol{b_1} \\ \boldsymbol{b_2} \end{bmatrix} + \begin{bmatrix} \boldsymbol{Z_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Z_2} \end{bmatrix} \begin{bmatrix} \boldsymbol{a_1} \\ \boldsymbol{a_2} \end{bmatrix} + \begin{bmatrix} \boldsymbol{e_1} \\ \boldsymbol{e_2} \end{bmatrix}$$

R.A. Mrode 2005 Linear Models for the Prediction of Animal Breeding Values

Copyright ©2020, Joanna Szyda

Wielocechowy model BLUP

$$\begin{bmatrix} \boldsymbol{y_1} \\ \boldsymbol{y_2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{X_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{X_2} \end{bmatrix} \begin{bmatrix} \boldsymbol{b_1} \\ \boldsymbol{b_2} \end{bmatrix} + \begin{bmatrix} \boldsymbol{Z_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Z_2} \end{bmatrix} \begin{bmatrix} \boldsymbol{a_1} \\ \boldsymbol{a_2} \end{bmatrix} + \begin{bmatrix} \boldsymbol{e_1} \\ \boldsymbol{e_2} \end{bmatrix}$$

- $\boldsymbol{y_i}$ obserwacje fenotypowe cechy i
- $\boldsymbol{b_i}$ estymatory efektów stałych cechy i
- $\boldsymbol{a_i}$ predyktory wartości hodowlanej cechy i
- $\boldsymbol{e_i}$ predyktory efektów błędu cechy i
- $\boldsymbol{X_i}$ macierz wystąpień dla klas efektów stałych
- $\boldsymbol{Z_i}$ macierz wystąpień dla klas efektów losowych

R.A. Mrode 2005 Linear Models for the Prediction of Animal Breeding Values

Copyright ©2021, Joanna Szyda

Wielocechowy model BLUP

$$\begin{bmatrix} \boldsymbol{y_1} \\ \boldsymbol{y_2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{X_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{X_2} \end{bmatrix} \begin{bmatrix} \boldsymbol{b_1} \\ \boldsymbol{b_2} \end{bmatrix} + \begin{bmatrix} \boldsymbol{Z_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Z_2} \end{bmatrix} \begin{bmatrix} \boldsymbol{a_1} \\ \boldsymbol{a_2} \end{bmatrix} + \begin{bmatrix} \boldsymbol{e_1} \\ \boldsymbol{e_2} \end{bmatrix}$$

- $var(\boldsymbol{a_i}) = \boldsymbol{A}\sigma_{a_i}^2 = \boldsymbol{A}g_{ii}$
- $cov(\boldsymbol{a_1}, \boldsymbol{a_2}) = \boldsymbol{A}\sigma_{a_{1,2}} = \boldsymbol{A}g_{12}$
- $var(\boldsymbol{e_i}) = \boldsymbol{I}\sigma_{e_i}^2 = \boldsymbol{I}r_{ii}$
- $cov(\boldsymbol{e_1}, \boldsymbol{e_2}) = \boldsymbol{I}\sigma_{e_{1,2}} = \boldsymbol{I}r_{12}$

R.A. Mrode 2005 Linear Models for the Prediction of Animal Breeding Values

Copyright ©2021, Joanna Szyda

Wielocechowy model BLUP

$$\begin{bmatrix} \boldsymbol{y_1} \\ \boldsymbol{y_2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{X_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{X_2} \end{bmatrix} \begin{bmatrix} \boldsymbol{b_1} \\ \boldsymbol{b_2} \end{bmatrix} + \begin{bmatrix} \boldsymbol{Z_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Z_2} \end{bmatrix} \begin{bmatrix} \boldsymbol{a_1} \\ \boldsymbol{a_2} \end{bmatrix} + \begin{bmatrix} \boldsymbol{e_1} \\ \boldsymbol{e_2} \end{bmatrix}$$

$$\boldsymbol{G_0} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \qquad \boldsymbol{R_0} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}$$

$$\boldsymbol{var} \begin{bmatrix} \boldsymbol{a_1} \\ \boldsymbol{a_2} \\ \boldsymbol{e_1} \\ \boldsymbol{e_2} \end{bmatrix} = \begin{bmatrix} g_{11}\boldsymbol{A} & g_{12}\boldsymbol{A} & \boldsymbol{0} & \boldsymbol{0} \\ g_{21}\boldsymbol{A} & g_{22}\boldsymbol{A} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & r_{11}\boldsymbol{I} & r_{12}\boldsymbol{I} \\ \boldsymbol{0} & \boldsymbol{0} & r_{21}\boldsymbol{I} & r_{22}\boldsymbol{I} \end{bmatrix}$$

R.A. Mrode 2005 Linear Models for the Prediction of Animal Breeding Values

Copyright ©2020, Joanna Szyda

Wielocechowy model BLUP

$$\begin{bmatrix} \boldsymbol{X}^T\boldsymbol{R}^{-1}\boldsymbol{X} & \boldsymbol{X}^T\boldsymbol{R}^{-1}\boldsymbol{Z} \\ \boldsymbol{Z}^T\boldsymbol{R}^{-1}\boldsymbol{X} & \boldsymbol{Z}^T\boldsymbol{R}^{-1}\boldsymbol{Z} + \boldsymbol{A}^{-1}\otimes\boldsymbol{G}^{-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{b} \\ \boldsymbol{a} \end{bmatrix} = \begin{bmatrix} \boldsymbol{X}^T\boldsymbol{R}^{-1}\boldsymbol{y} \\ \boldsymbol{Z}^T\boldsymbol{R}^{-1}\boldsymbol{y} \end{bmatrix}$$

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{X}_1 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{X}_2 \end{bmatrix} \quad \boldsymbol{Z} = \begin{bmatrix} \boldsymbol{Z}_1 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Z}_2 \end{bmatrix} \quad \boldsymbol{b} = \begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \end{bmatrix} \quad \boldsymbol{a} = \begin{bmatrix} \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \end{bmatrix}$$

$$\boldsymbol{G}_0^{-1} = \begin{bmatrix} g^{11} & g^{12} \\ g^{21} & g^{22} \end{bmatrix} \quad \boldsymbol{R}_0^{-1} = \begin{bmatrix} r^{11} & r^{12} \\ r^{21} & r^{22} \end{bmatrix} \quad \boldsymbol{R}^{11} = \boldsymbol{I}r^{11}$$

$$\begin{bmatrix} \hat{\boldsymbol{b}}_1 \\ \hat{\boldsymbol{b}}_2 \\ \hat{\boldsymbol{a}}_1 \\ \hat{\boldsymbol{a}}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{X}_1^T\boldsymbol{R}^{11}\boldsymbol{X}_1 & \boldsymbol{X}_1^T\boldsymbol{R}^{12}\boldsymbol{X}_2 & \boldsymbol{X}_1^T\boldsymbol{R}^{11}\boldsymbol{Z}_1 + & \boldsymbol{X}_1^T\boldsymbol{R}^{12}\boldsymbol{Z}_2 + \\ \boldsymbol{X}_2^T\boldsymbol{R}^{12}\boldsymbol{X}_1 & \boldsymbol{X}_2^T\boldsymbol{R}^{22}2 & \boldsymbol{X}_2^T\boldsymbol{R}^{21}\boldsymbol{Z}_1 + & \boldsymbol{X}_2^T\boldsymbol{R}^{22}\boldsymbol{Z}_2 + \\ \boldsymbol{Z}_1^T\boldsymbol{R}^{11}\boldsymbol{X}_1 & \boldsymbol{Z}_1^T\boldsymbol{R}^{12}\boldsymbol{X}_2 & \boldsymbol{Z}_1^T\boldsymbol{R}^{11}\boldsymbol{Z}_1 + \boldsymbol{A}^{-1}g^{11} & \boldsymbol{Z}_1^T\boldsymbol{R}^{12}\boldsymbol{Z}_2 + \boldsymbol{A}^{-1}g^{12} \\ \boldsymbol{Z}_2^T\boldsymbol{R}^{21}\boldsymbol{X}_1 & \boldsymbol{Z}_2^T\boldsymbol{R}^{22}\boldsymbol{X}_2 & \boldsymbol{Z}_2^T\boldsymbol{R}^{21}\boldsymbol{Z}_1 + \boldsymbol{A}^{-1}g^{21} & \boldsymbol{Z}_2^T\boldsymbol{R}^{22}\boldsymbol{Z}_2 + \boldsymbol{A}^{-1}g^{22} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{X}_1^T\boldsymbol{R}^{11}\boldsymbol{y}_1 + \boldsymbol{X}_1^T\boldsymbol{R}^{12}\boldsymbol{y}_2 \\ \boldsymbol{X}_2^T\boldsymbol{R}^{21}\boldsymbol{y}_1 + \boldsymbol{X}_2^T\boldsymbol{R}^{22}\boldsymbol{y}_2 \\ \boldsymbol{Z}_1^T\boldsymbol{R}^{11}\boldsymbol{y}_1 + \boldsymbol{Z}_1^T\boldsymbol{R}^{12}\boldsymbol{y}_2 \\ \boldsymbol{Z}_2^T\boldsymbol{R}^{21}\boldsymbol{y}_1 + \boldsymbol{Z}_2^T\boldsymbol{R}^{22}\boldsymbol{y}_2 \end{bmatrix}$$

R.A. Mrode 2005 Linear Models for the Prediction of Animal Breeding Values

Copyright ©2023, Joanna Szyda

Identyczne macierze wystąpień

Identyczne macierze wystąpień

Dane

Calf	Sex	Sire	Dam	WWG	PWG
4	Male	1	–	4.5	6.8
5	Female	3	2	2.9	5.0
6	Female	1	2	3.9	6.8
7	Male	4	5	3.5	6.0
8	Male	3	6	5.0	7.5

WWG, pre-weaning gain; PWG, post-weaning gain.

Model

$$\begin{bmatrix} \boldsymbol{y_1} \\ \boldsymbol{y_2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{X_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{X_2} \end{bmatrix} \begin{bmatrix} \boldsymbol{b_1} \\ \boldsymbol{b_2} \end{bmatrix} + \begin{bmatrix} \boldsymbol{Z_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Z_2} \end{bmatrix} \begin{bmatrix} \boldsymbol{a_1} \\ \boldsymbol{a_2} \end{bmatrix} + \begin{bmatrix} \boldsymbol{e_1} \\ \boldsymbol{e_2} \end{bmatrix}$$

Macierz wystąpień

$$\boldsymbol{X_1^T} = \boldsymbol{X_2^T} = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

LHS

$$\boldsymbol{y^T} = \begin{bmatrix} \boldsymbol{y_1^T} & \boldsymbol{y_2^T} \end{bmatrix} = [4.5 \;\; 2.9 \;\; 3.9 \;\; 3.5 \;\; 5.0 \;\; 6.8 \;\; 5.0 \;\; 6.0 \;\; 6.0 \;\; 7.5\,]$$

R.A. Mrode 2005 Linear Models for the Prediction of Animal Breeding Values

Copyright ©2022, Joanna Szyda

Identyczne macierze wystąpień

Struktura kowariancji

$$\boldsymbol{G_0} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = \begin{bmatrix} 20 & 18 \\ 18 & 40 \end{bmatrix}$$

$$\boldsymbol{R_0} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} = \begin{bmatrix} 40 & 11 \\ 11 & 30 \end{bmatrix}$$

$$\boldsymbol{var}\begin{bmatrix} \boldsymbol{a_1} \\ \boldsymbol{a_2} \\ \boldsymbol{e_1} \\ \boldsymbol{e_2} \end{bmatrix} = \begin{bmatrix} g_{11}\boldsymbol{A} & g_{12}\boldsymbol{A} & \boldsymbol{0} & \boldsymbol{0} \\ g_{21}\boldsymbol{A} & g_{22}\boldsymbol{A} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & r_{11}\boldsymbol{I} & r_{12}\boldsymbol{I} \\ \boldsymbol{0} & \boldsymbol{0} & r_{21}\boldsymbol{I} & r_{22}\boldsymbol{I} \end{bmatrix} = \begin{bmatrix} g_{11}\boldsymbol{A} & g_{12}\boldsymbol{A} & \boldsymbol{0} & \boldsymbol{0} \\ g_{21}\boldsymbol{A} & g_{22}\boldsymbol{A} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{R}_{11} & \boldsymbol{R}_{12} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{R}_{21} & \boldsymbol{R}_{22} \end{bmatrix}$$

R.A. Mrode 2005 Linear Models for the Prediction of Animal Breeding Values

Copyright ©2022, Joanna Szyda

Identyczne macierze wystąpień

$$\boldsymbol{G}_0^{-1} = \begin{bmatrix} g^{11} & g^{12} \\ g^{21} & g^{22} \end{bmatrix} = \begin{bmatrix} 0.084 & -0.038 \\ -0.038 & 0.042 \end{bmatrix}$$

$$\boldsymbol{R}_0^{-1} = \begin{bmatrix} r^{11} & r^{12} \\ r^{21} & r^{22} \end{bmatrix} = \begin{bmatrix} 0.028 & -0.010 \\ -0.010 & 0.037 \end{bmatrix}$$

$$\begin{bmatrix} \hat{\boldsymbol{b}}_1 \\ \hat{\boldsymbol{b}}_2 \\ \hat{\boldsymbol{a}}_1 \\ \hat{\boldsymbol{a}}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{X_1^T R^{11} X_1} & \boldsymbol{X_1^T R^{12} X_2} & \boldsymbol{X_1^T R^{11} Z_1} + & \boldsymbol{X_1^T R^{12} Z_2} + \\ \boldsymbol{X_2^T R^{12} X_1} & \boldsymbol{X_2^T R^{22} 2} & \boldsymbol{X_2^T R^{21} Z_1} + & \boldsymbol{X_2^T R^{22} Z_2} + \\ \boldsymbol{Z_1^T R^{11} X_1} & \boldsymbol{Z_1^T R^{12} X_2} & \boldsymbol{Z_1^T R^{11} Z_1} + \boldsymbol{A^{-1}} g^{11} & \boldsymbol{Z_1^T R^{12} Z_2} + \boldsymbol{A^{-1}} g^{12} \\ \boldsymbol{Z_2^T R^{21} X_1} & \boldsymbol{Z_2^T R^{22} X_2} & \boldsymbol{Z_2^T R^{21} Z_1} + \boldsymbol{A^{-1}} g^{21} & \boldsymbol{Z_2^T R^{22} Z_2} + \boldsymbol{A^{-1}} g^{22} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{X_1^T R^{11} y_1} + \boldsymbol{X_1^T R^{12} y_2} \\ \boldsymbol{X_2^T R^{21} y_1} + \boldsymbol{X_2^T R^{22} y_2} \\ \boldsymbol{Z_1^T R^{11} y_1} + \boldsymbol{Z_1^T R^{12} y_2} \\ \boldsymbol{Z_2^T R^{21} y_1} + \boldsymbol{Z_2^T R^{22} y_2} \end{bmatrix}$$

R.A. Mrode 2005 Linear Models for the Prediction of Animal Breeding Values

Copyright ©2020, Joanna Szyda

Identyczne macierze wystąpień

Rozwiązania

	Multivariate analysis traits		Univariate analysis traits	
Effects	WWG	PWG	WWG	PWG
*Sex**				
1	4.361	6.800	4.358	6.798
2	3.397	5.880	3.404	5.879
Animal				
1	0.151	0.280	0.098	0.277
2	–0.015	–0.008	–0.019	–0.005
3	–0.078	–0.170	–0.041	–0.171
4	–0.010	–0.013	–0.009	–0.013
5	–0.270	–0.478	–0.186	–0.471
6	0.276	0.517	0.177	0.514
7	–0.316	–0.479	–0.249	–0.464
8	0.244	0.392	0.183	0.384

model jedno vs wielocechowy

- Podobne różnice dla płci
- Małe różnice dla PWG → wysoka odziedziczalność 0.57
- Większe różnice dla WWG → niższa odziedziczalność 0.33

R.A. Mrode 2005 Linear Models for the Prediction of Animal Breeding Values

Copyright ©2020, Joanna Szyda

Identyczne macierze wystąpień

Dokładność predykcji wartości hodowlanej

$$\begin{bmatrix} \boldsymbol{X}^T\boldsymbol{X} & \boldsymbol{X}^T\boldsymbol{Z} \\ \boldsymbol{Z}^T\boldsymbol{X} & \boldsymbol{Z}^T\boldsymbol{Z} + \boldsymbol{A}^{-1}\sigma_a^2 \end{bmatrix}^{-1} = \begin{bmatrix} C^{11} & C^{12} \\ C^{21} & C^{22} \end{bmatrix}$$

$$\left[\begin{array}{cc|cccccccc}
0.596 & 0.157 & -0.164 & -0.084 & -0.131 & -0.265 & -0.148 & -0.166 & -0.284 & -0.238 \\
0.157 & 0.802 & -0.133 & -0.241 & -0.112 & -0.087 & -0.299 & -0.306 & -0.186 & -0.199 \\
\hline
-0.164 & -0.133 & 0.471 & 0.007 & 0.033 & 0.220 & 0.045 & 0.221 & 0.139 & 0.134 \\
-0.084 & -0.241 & 0.007 & 0.492 & -0.010 & 0.020 & 0.237 & 0.245 & 0.120 & 0.111 \\
-0.131 & -0.112 & 0.033 & -0.010 & 0.456 & 0.048 & 0.201 & 0.023 & 0.126 & 0.218 \\
-0.265 & -0.087 & 0.220 & 0.020 & 0.048 & 0.428 & 0.047 & 0.128 & 0.243 & 0.123 \\
-0.148 & -0.299 & 0.045 & 0.237 & 0.201 & 0.047 & 0.428 & 0.170 & 0.220 & 0.178 \\
-0.166 & -0.306 & 0.221 & 0.245 & 0.023 & 0.128 & 0.170 & 0.442 & 0.152 & 0.219 \\
-0.284 & -0.186 & 0.139 & 0.120 & 0.126 & 0.243 & 0.220 & 0.152 & 0.442 & 0.168 \\
-0.238 & -0.199 & 0.134 & 0.111 & 0.218 & 0.123 & 0.178 & 0.219 & 0.168 & 0.422
\end{array}\right]$$

$$\begin{bmatrix} \hat{\boldsymbol{b}}_1 \\ \hat{\boldsymbol{b}}_2 \\ \hat{\boldsymbol{a}}_1 \\ \hat{\boldsymbol{a}}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{X}_1^T\boldsymbol{R}^{11}\boldsymbol{X}_1 & \boldsymbol{X}_1^T\boldsymbol{R}^{12}\boldsymbol{X}_2 & \boldsymbol{X}_1^T\boldsymbol{R}^{11}\boldsymbol{Z}_1 + & \boldsymbol{X}_1^T\boldsymbol{R}^{12}\boldsymbol{Z}_2 + \\ \boldsymbol{X}_2^T\boldsymbol{R}^{12}\boldsymbol{X}_1 & \boldsymbol{X}_2^T\boldsymbol{R}^{22}\boldsymbol{2} & \boldsymbol{X}_2^T\boldsymbol{R}^{21}\boldsymbol{Z}_1 + & \boldsymbol{X}_2^T\boldsymbol{R}^{22}\boldsymbol{Z}_2 + \\ \boldsymbol{Z}_1^T\boldsymbol{R}^{11}\boldsymbol{X}_1 & \boldsymbol{Z}_1^T\boldsymbol{R}^{12}\boldsymbol{X}_2 & \boldsymbol{Z}_1^T\boldsymbol{R}^{11}\boldsymbol{Z}_1 + \boldsymbol{A}^{-1}g^{11} & \boldsymbol{Z}_1^T\boldsymbol{R}^{12}\boldsymbol{Z}_2 + \boldsymbol{A}^{-1}g^{12} \\ \boldsymbol{Z}_2^T\boldsymbol{R}^{21}\boldsymbol{X}_1 & \boldsymbol{Z}_2^T\boldsymbol{R}^{22}\boldsymbol{X}_2 & \boldsymbol{Z}_2^T\boldsymbol{R}^{21}\boldsymbol{Z}_1 + \boldsymbol{A}^{-1}g^{21} & \boldsymbol{Z}_2^T\boldsymbol{R}^{22}\boldsymbol{Z}_2 + \boldsymbol{A}^{-1}g^{22} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{X}_1^T\boldsymbol{R}^{11}\boldsymbol{y}_1 + \boldsymbol{X}_1^T\boldsymbol{R}^{12}\boldsymbol{y}_2 \\ \boldsymbol{X}_2^T\boldsymbol{R}^{21}\boldsymbol{y}_1 + \boldsymbol{X}_2^T\boldsymbol{R}^{22}\boldsymbol{y}_2 \\ \boldsymbol{Z}_1^T\boldsymbol{R}^{11}\boldsymbol{y}_1 + \boldsymbol{Z}_1^T\boldsymbol{R}^{12}\boldsymbol{y}_2 \\ \boldsymbol{Z}_2^T\boldsymbol{R}^{21}\boldsymbol{y}_1 + \boldsymbol{Z}_2^T\boldsymbol{R}^{22}\boldsymbol{y}_2 \end{bmatrix}$$

R.A. Mrode 2005 Linear Models for the Prediction of Animal Breeding Values

Copyright ©2023, Joanna Szyda

Identyczne macierze wystąpień

Dokładność predykcji wartości hodowlanej

	Multivariate analysis				Univariate analysis reliability	
	Diagonals[a]		Reliability			
Animal	WWG	PWG	WWG	PWG	WWG	PWG
1	18.606	35.904	0.070	0.102	0.058	0.102
2	19.596	38.768	0.020	0.031	0.016	0.031
3	17.893	33.799	0.105	0.155	0.088	0.155
4	16.506	29.727	0.175	0.257	0.144	0.256
5	16.541	29.865	0.173	0.253	0.144	0.253
6	17.152	31.504	0.142	0.212	0.116	0.212
7	17.115	31.364	0.144	0.216	0.116	0.216
8	16.285	29.160	0.186	0.271	0.156	0.270

$$r_i = r(\hat{a}_i, a_i) = \sqrt{1 - c_i^{22}\frac{\sigma_e^2}{\sigma_a^2}}$$

- Podobna dokładność dla PWG → wyższa h^2
- Wzrost dokładności dla WWG → niższa h^2
- Dodatkowa informacja z $\boldsymbol{G}$ i $\boldsymbol{R}_0$

R.A. Mrode 2005 Linear Models for the Prediction of Animal Breeding Values

Copyright ©2020, Joanna Szyda

Brakujące rekordy danych

Brakujące rekordy danych

Dane

Calf	Sex	Sire	Dam	WWG (kg)	PWG (kg)
4	Male	1	–	4.5	–
5	Female	3	2	2.9	5.0
6	Female	1	2	3.9	6.8
7	Male	4	5	3.5	6.0
8	Male	3	6	5.0	7.5
9	Female	7	8	4.0	–

Brakujące dane dla PWG → osobniki 4 i 9

R.A. Mrode 2005 Linear Models for the Prediction of Animal Breeding Values

Copyright ©2020, Joanna Szyda

Brakujące rekordy danych

$$\begin{bmatrix} \boldsymbol{y_1} \\ \boldsymbol{y_2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{X_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{X_2} \end{bmatrix} \begin{bmatrix} \boldsymbol{b_1} \\ \boldsymbol{b_2} \end{bmatrix} + \begin{bmatrix} \boldsymbol{Z_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Z_2} \end{bmatrix} \begin{bmatrix} \boldsymbol{a_1} \\ \boldsymbol{a_2} \end{bmatrix} + \begin{bmatrix} \boldsymbol{e_1} \\ \boldsymbol{e_2} \end{bmatrix}$$

$$\boldsymbol{X}_1 = \begin{bmatrix} 10 \\ 01 \\ 01 \\ 10 \\ 10 \\ 01 \end{bmatrix} \quad \boldsymbol{X}_2 = \begin{bmatrix} 00 \\ 01 \\ 01 \\ 10 \\ 10 \\ 00 \end{bmatrix}$$

R.A. Mrode 2005 Linear Models for the Prediction of Animal Breeding Values

Copyright ©2020, Joanna Szyda

Brakujące rekordy danych

$$\begin{bmatrix}\hat{b}_1\\ \hat{b}_2\\ \hat{a}_1\\ \hat{a}_2\end{bmatrix}=\begin{bmatrix}X_1^TR^{11}X_1 & X_1^TR^{12}X_2 & X_1^TR^{11}Z_1 & X_1^TR^{12}Z_2\\ X_2^TR^{12}X_1 & X_2^TR^{22}2 & X_2^TR^{21}Z_1 & X_2^TR^{22}Z_2\\ Z_1^TR^{11}X_1 & Z_1^TR^{12}X_2 & Z_1^TR^{11}Z_1+A^{-1}g^{11} & Z_1^TR^{12}Z_2+A^{-1}g^{12}\\ Z_2^TR^{21}X_1 & Z_2^TR^{22}X_2 & Z_2^TR^{21}Z_1+A^{-1}g^{21} & Z_2^TR^{22}Z_2+A^{-1}g^{22}\end{bmatrix}^{-1}\begin{bmatrix}X_1^TR^{11}y_1+X_1^TR^{12}y_2\\ X_2^TR^{21}y_1+X_2^TR^{22}y_2\\ Z_1^TR^{11}y_1+Z_1^TR^{12}y_2\\ Z_2^TR^{21}y_1+Z_2^TR^{22}y_2\end{bmatrix}$$

- Różne macierze kowariancji błędu $\boldsymbol{R}^{11}=\boldsymbol{I}r^{11}$

  - osobniki z brakującymi danymi → odwrotność wartości wariancji

$$\boldsymbol{R}_o^{-1}=\begin{bmatrix}\frac{1}{40} & -0.010\\ -0.010 & 0.037\end{bmatrix}$$

  - osobniki z pełnymi danymi → element odwróconej macierzy $\boldsymbol{R}_o$

$$\boldsymbol{R}_o^{-1}=\begin{bmatrix}0.028 & -0.010\\ -0.010 & 0.037\end{bmatrix}$$

R.A. Mrode 2005 Linear Models for the Prediction of Animal Breeding Values

Copyright ©2023, Joanna Szyda

Brakujące rekordy danych

$$r_m^{11} = \frac{1}{40} \qquad r_o^{11} = 0.028$$

$$\begin{bmatrix} \hat{\boldsymbol{b}}_1 \\ \hat{\boldsymbol{b}}_2 \\ \hat{\boldsymbol{a}}_1 \\ \hat{\boldsymbol{a}}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{X}_1^T\boldsymbol{R}^{11}\boldsymbol{X}_1 & \boldsymbol{X}_1^T\boldsymbol{R}^{12}\boldsymbol{X}_2 & \boldsymbol{X}_1^T\boldsymbol{R}^{11}\boldsymbol{Z}_1 & \boldsymbol{X}_1^T\boldsymbol{R}^{12}\boldsymbol{Z}_2 \\ \boldsymbol{X}_2^T\boldsymbol{R}^{12}\boldsymbol{X}_1 & \boldsymbol{X}_2^T\boldsymbol{R}^{22}\boldsymbol{X}_2 & \boldsymbol{X}_2^T\boldsymbol{R}^{21}\boldsymbol{Z}_1 & \boldsymbol{X}_2^T\boldsymbol{R}^{22}\boldsymbol{Z}_2 \\ \boldsymbol{Z}_1^T\boldsymbol{R}^{11}\boldsymbol{X}_1 & \boldsymbol{Z}_1^T\boldsymbol{R}^{12}\boldsymbol{X}_2 & \boldsymbol{Z}_1^T\boldsymbol{R}^{11}\boldsymbol{Z}_1 + \boldsymbol{A}^{-1}g^{11} & \boldsymbol{Z}_1^T\boldsymbol{R}^{12}\boldsymbol{Z}_2 + \boldsymbol{A}^{-1}g^{12} \\ \boldsymbol{Z}_2^T\boldsymbol{R}^{21}\boldsymbol{X}_1 & \boldsymbol{Z}_2^T\boldsymbol{R}^{22}\boldsymbol{X}_2 & \boldsymbol{Z}_2^T\boldsymbol{R}^{21}\boldsymbol{Z}_1 + \boldsymbol{A}^{-1}g^{21} & \boldsymbol{Z}_2^T\boldsymbol{R}^{22}\boldsymbol{Z}_2 + \boldsymbol{A}^{-1}g^{22} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{X}_1^T\boldsymbol{R}^{11}\boldsymbol{y}_1 + \boldsymbol{X}_1^T\boldsymbol{R}^{12}\boldsymbol{y}_2 \\ \boldsymbol{X}_2^T\boldsymbol{R}^{21}\boldsymbol{y}_1 + \boldsymbol{X}_2^T\boldsymbol{R}^{22}\boldsymbol{y}_2 \\ \boldsymbol{Z}_1^T\boldsymbol{R}^{11}\boldsymbol{y}_1 + \boldsymbol{Z}_1^T\boldsymbol{R}^{12}\boldsymbol{y}_2 \\ \boldsymbol{Z}_2^T\boldsymbol{R}^{21}\boldsymbol{y}_1 + \boldsymbol{Z}_2^T\boldsymbol{R}^{22}\boldsymbol{y}_2 \end{bmatrix}$$

Zależnie od osobnika $\boldsymbol{R}^{11}$ zawiera na przekątnej r_m^{11} lub r_o^{11}

R.A. Mrode 2005 Linear Models for the Prediction of Animal Breeding Values

Copyright ©2023, Joanna Szyda

Brakujące rekordy danych

Rozwiązania

	Multivariate analysis		Univariate analysis	
Effects	WWG	PWG	WWG	PWG
Sex				
1	4.327	6.793	4.325	6.784
2	3.598	5.966	3.599	5.873
Animal				
1	0.154	0.288	0.100	0.273
2	−0.059	−0.053	−0.059	0.000
3	−0.061	−0.163	−0.022	−0.165
4	0.027	0.036	0.022	−0.025
5	−0.307	−0.521	−0.218	−0.464
6	0.235	0.477	0.138	0.518
7	−0.280	−0.452	−0.211	−0.460
8	0.272	0.407	0.214	0.392
9	0.077	0.051	0.081	−0.034

model jedno- vs wielo-cechowy

- Różnice w estymatorach płci dla PWG
- Różnice w wartościach hodowlanych PWG dla osobników 4 i 9 (brakujące rekordy PWG)

→ model jednocechowy: informacja od krewnych

→ model wielocechowy: informacja od krewnych + ***G*** + ***R***

R.A. Mrode 2005 Linear Models for the Prediction of Animal Breeding Values

Copyright ©2023, Joanna Szyda

Różne macierze wystąpień

Różne macierze wystąpień

Dane

Cow	Sire	Dam	HYS1	HYS2	FAT1	FAT2
4	1	2	1	1	201	280
5	3	2	1	2	150	200
6	1	5	2	1	160	190
7	3	4	1	1	180	250
8	1	7	2	2	285	300

HYS1, HYS2, herd–year–season for parity 1 and 2, respectively; FAT1, FAT2, fat yield in parity 1 and 2.

Różne wartości efektu stałego HYS (herd-year-season_of_calving) dla cech FAT1 i FAT2

$$\boldsymbol{X}_1^T = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \qquad \boldsymbol{X}_2^T = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

R.A. Mrode 2005 Linear Models for the Prediction of Animal Breeding Values

Copyright ©2020, Joanna Szyda

Różne macierze wystąpień

$$\begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \hat{a}_1 \\ \hat{a}_2 \end{bmatrix} = \begin{bmatrix} X_1^T R^{11} X_1 & X_1^T R^{12} X_2 & X_1^T R^{11} Z_1 + & X_1^T R^{12} Z_2 + \\ X_2^T R^{12} X_1 & X_2^T R^{22} 2 & X_2^T R^{21} Z_1 + & X_2^T R^{22} Z_2 + \\ Z_1^T R^{11} X_1 & Z_1^T R^{12} X_2 & Z_1^T R^{11} Z_1 + A^{-1} g^{11} & Z_1^T R^{12} Z_2 + A^{-1} g^{12} \\ Z_2^T R^{21} X_1 & Z_2^T R^{22} X_2 & Z_2^T R^{21} Z_1 + A^{-1} g^{21} & Z_2^T R^{22} Z_2 + A^{-1} g^{22} \end{bmatrix}^{-1} \begin{bmatrix} X_1^T R^{11} y_1 + X_1^T R^{12} y_2 \\ X_2^T R^{21} y_1 + X_2^T R^{22} y_2 \\ Z_1^T R^{11} y_1 + Z_1^T R^{12} y_2 \\ Z_2^T R^{21} y_1 + Z_2^T R^{22} y_2 \end{bmatrix}$$

Macierz wystąpień $X_1 \neq X_2 \quad Z_1 = Z_2$

R.A. Mrode 2005 Linear Models for the Prediction of Animal Breeding Values

Copyright ©2023, Joanna Szyda

Różne macierze wystąpień

Rozwiązania

	Multivariate analysis		Univariate analysis	
Effects	FAT1	FAT2	FAT1	FAT2
HYS				
1	175.7	243.2	175.8	237.1
2	219.6	240.6	220.4	250.0
Animal				
1	8.969	8.840	6.933	8.665
2	−2.999	−2.777	−2.590	−2.244
3	−5.970	−6.063	−4.341	−6.422
4	11.754	11.658	9.103	12.197
5	−16.253	−15.824	−12.992	−15.563
6	−17.314	−15.719	−15.197	−11.149
7	8.690	8.138	7.566	7.696
8	22.702	20.931	19.417	15.560

$$\boldsymbol{G_0} = \begin{bmatrix} 35 & 28 \\ 28 & 30 \end{bmatrix}$$

$$\boldsymbol{R_0} = \begin{bmatrix} 65 & 27 \\ 27 & 70 \end{bmatrix}$$

$$h^2_{FAT1} = \frac{35}{100}$$

$$h^2_{FAT2} = \frac{30}{100}$$

R.A. Mrode 2005 Linear Models for the Prediction of Animal Breeding Values

Copyright ©2021, Joanna Szyda

MACE

MACE

MACE model dla pojedynczej cechy np. wydajność mleka w kraju i

$$\boldsymbol{y_i = \mu_i + Z_i Q w_i + Z_i a_i + e_i}$$

- $\boldsymbol{y_i}$ DRP buhaja w kraju i
- $\boldsymbol{\mu_i}$ średnia dla kraju i
- $\boldsymbol{w_i}$ efekt grupy genetycznej dla kraju i
- $\boldsymbol{a_i}$ wartość hodowlana buhaja w kraju i
- $\boldsymbol{e_i}$ predyktory efektów błędu dla kraju i
- $\boldsymbol{Q_i}$ macierz wystąpień dla klas fantom parents
- $\boldsymbol{Z_i}$ macierz wystąpień dla wartości hodowlanych

R.A. Mrode 2005 Linear Models for the Prediction of Animal Breeding Values

Copyright ©2022, Joanna Szyda

MACE

MACE model dla pojedynczej cechy dla krajów 1 i 2

$$var\begin{bmatrix} \boldsymbol{w_1} \\ \boldsymbol{w_2} \\ \boldsymbol{a_1} \\ \boldsymbol{a_2} \\ \boldsymbol{e_1} \\ \boldsymbol{e_2} \end{bmatrix} = \left[\begin{array}{cc|cc|cc} g_{11}\boldsymbol{A}_p & g_{12}\boldsymbol{A}_p & g_{11}\boldsymbol{A}_{pa} & g_{12}\boldsymbol{A}_{pa} & \boldsymbol{0} & \boldsymbol{0} \\ g_{12}\boldsymbol{A}_p & g_{22}\boldsymbol{A}_p & g_{12}\boldsymbol{A}_{pa} & g_{22}\boldsymbol{A}_{pa} & \boldsymbol{0} & \boldsymbol{0} \\ \hline g_{11}\boldsymbol{A}_{pa} & g_{12}\boldsymbol{A}_{pa} & g_{11}\boldsymbol{A}_a & g_{12}\boldsymbol{A}_a & \boldsymbol{0} & \boldsymbol{0} \\ g_{12}\boldsymbol{A}_{pa} & g_{22}\boldsymbol{A}_{pa} & g_{12}\boldsymbol{A}_a & g_{22}\boldsymbol{A}_a & \boldsymbol{0} & \boldsymbol{0} \\ \hline \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{D}_1\sigma_{e1}^2 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{D}_2\sigma_{e2}^2 \end{array}\right]$$

- $\boldsymbol{A_p}$ macierz spokrewnień pomiędzy grupami genetycznymi
- $\boldsymbol{A_a}$ macierz spokrewnień pomiędzy buhajami
- $\boldsymbol{A_{pa}}$ macierz spokrewnień pomiędzy **w** i **a**
- $\boldsymbol{D_i}$ macierz diagonalna z $^{\boldsymbol{1}}/_{\boldsymbol{EDC}}$ dla kraju i
- $\sigma_{\boldsymbol{ei}}^{\boldsymbol{2}}$ wariancja błędu dla kraju

R.A. Mrode 2005 Linear Models for the Prediction of Animal Breeding Values

Copyright ©2022, Joanna Szyda

MACE przykładowe dane

	Country 1			Country 2		
Sire	EDC	BV	DRP	EDC	BV	DRP
1	58	9.0	9.7229	90	13.5	14.5088
2	150	10.1	9.9717	65	7.6	7.7594
3	20	15.8	19.2651	–	–	–
4	25	–4.7	–8.5711	–	–	–
5	–	–	–	30	19.6	23.9672
6	–	–	–	55	–5.3	–9.6226

EDC, effective daughter contribution; BV, breeding value; DRP, deregressed proof.

R.A. Mrode 2005 Linear Models for the Prediction of Animal Breeding Values

Copyright ©2020, Joanna Szyda

MACE przykładowa struktura spokrewnienia

Bull	Sire	MGS	MGD
1	7	G3	G5
2	8	9	G5
3	7	2	G5
4	1	G2	G5
5	8	G3	G4
6	1	9	G4
7	G1	G2	G4
8	G1	G2	G4
9	G1	G2	G4

R.A. Mrode 2005 Linear Models for the Prediction of Animal Breeding Values

Copyright ©2020, Joanna Szyda

MACE przykładowe wyniki dla krajów 1 i 2

Effects	Solutions			
	Country 1		Country 2	
Country effect				
	7.268		9.036	
Animal/group				
	A	B	A	B
1	2.604	9.871	2.661	11.697
2	2.176	9.444	0.403	9.439
3	8.059	15.327	5.001	14.037
4	–9.865	–2.597	–5.605	3.431
5	13.634	20.902	9.728	18.764
6	–18.086	–10.818	–13.203	–4.167
7	4.310	11.578	3.071	12.106
8	7.015	14.283	4.489	13.525
9	–6.299	0.969	–5.059	3.977
G1	0.174	7.442	–0.092	8.944
G2	–0.124	7.144	0.126	9.162
G3	–0.071	7.197	0.264	9.300
G4	0.087	7.355	–0.288	8.748
G5	–0.067	7.201	–0.010	9.026

A = solutions for animals and groups from the MME; B = solutions for animals and groups expressed in each country scale.

Copyright ©2020, Joanna Szyda

literatura

Journal of ANIMAL SCIENCE

Issues | More Content ▾ | Submit ▾ | Purchase | Alerts | About ▾ | Journal of Animal S

Volume 97, Issue 1
January 2019

JOURNAL ARTICLE

Multivariate genomic predictions for age at puberty in tropically adapted beef heifers[1] FREE

Bailey N Engle, Nicholas J Corbet, Jamie M Allen, Alan R Laing, Geoffry Fordyce, Michael R McGowan, Brian M Burns, Russell E Lyons, Ben J Hayes

Journal of Animal Science, Volume 97, Issue 1, January 2019, Pages 90–100,
https://doi.org/10.1093/jas/sky428
Published: 09 November 2

$$\begin{bmatrix} y_{\mathrm{AGECL}} \\ y_{\mathrm{RMS}} \end{bmatrix} = \begin{bmatrix} X_{\mathrm{AGECL}} & 0 \\ 0 & X_{\mathrm{RMS}} \end{bmatrix} \begin{bmatrix} \beta_{\mathrm{AGECL}} \\ \beta_{\mathrm{RMS}} \end{bmatrix} + \begin{bmatrix} M_{\mathrm{AGECL}} & 0 \\ 0 & M_{\mathrm{RMS}} \end{bmatrix} \begin{bmatrix} u_{\mathrm{AGECL}} \\ u_{\mathrm{RMS}} \end{bmatrix} + \begin{bmatrix} \varepsilon_{\mathrm{AGECL}} \\ \varepsilon_{\mathrm{RMS}} \end{bmatrix}$$

Copyright ©2023, Joanna Szyda

literatura

ELSEVIER

Journal of Dairy Science

Volume 104, Issue 10, October 2021, Pages 10905-10920

Research

Including milk production, conformation, and functional traits in multivariate models for genetic evaluation of lameness

M. Khansefid [1], M. Haile-Mariam [1], J.E. Pryce [1 2]

Show more

+ Add to Mendeley Share Cite

$$\begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} X_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & X_n \end{pmatrix} \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} + \begin{pmatrix} Z_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & Z_n \end{pmatrix} \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix} + \begin{pmatrix} w_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & w_n \end{pmatrix} \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix} + \begin{pmatrix} e_1 \\ \vdots \\ e_n \end{pmatrix},$$

Copyright ©2023, Joanna Szyda

1. Wielocechowy model BLUP
2. Identyczne macierze wystąpień
3. Brakujące rekordy danych
4. Różne macierze wystąpień
5. MACE

ELSEVIER

Journal of Dairy Science
Volume 77, Issue 9, September 1994, Pages 2671-2678

1. Co oznacza skrót MACE ?
2. Który model ma większe rozmiary – jedno czy wielo cechowy?
3. Na czy polega zaleta stosowania modelu wielocechowego?

MACE

Effective **D**aughter **C**ontributions

$$EDC_i = \sum_{k=1}^{N} \frac{\lambda r_k}{4 - r_k(1 + r_{dk})}$$

- k k-ta córka buhaja i
- $\lambda = \frac{4-h^2}{h^2}$
- r_k dokładność wydajności córki k: $r_k = \frac{n_k h^2}{1+(n_k-1)r}$
- r_{dk} dokładność wydajności matki córki k
- n_k liczba laktacji córki k
- h^2 odziedziczalność cechy, r powtarzalność (między laktacjami)

R.A. Mrode 2005 Linear Models for the Prediction of Animal Breeding Values

Copyright ©2020, Joanna Szyda