Zaawansowane aspekty praktycznego zastosowania modeli mieszanych

1. Wykład wstępny
2. Algebra macierzowa
3. Best Linear Unbiased Prediction (BLUP): model jednocechowy z pojedynczym efektem losowym
4. Best Linear Unbiased Prediction (BLUP): model wielocechowy

5. Metody redukcji wymiarów modeli wielocechowych BLUP

6. Analiza danych powtarzalnych w czasie
7. Predykcja genomowej wartości hodowlanej
8. Model jednostopniowy
9. Estymacja parametrów modelu
10. - 15. Prezentacje zaliczeniowe

1. Transformacja kanoniczna
2. Transformacja Choleskiego

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- Wielocechowy model BLUP
  - duże wymiary modelu
  - estymacja parametrów wymagająca obliczeniowo

- Metody redukcji wymiarów modelu
  - Transformacja kanoniczna ***canonical transformation*** → jednakowe macierze wystąpień, bez brakujących danych
  - Transformacja Choleskiego ***Cholesky transformation*** → sekwencyjnie brakujące dane

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Wymiary modelu

Wymiary modelu – łatwość wycieleń → 04.2020 04.2022 12.2023

$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{b} + \boldsymbol{Z}\boldsymbol{a} + \boldsymbol{e}$$

- $\boldsymbol{y}$ obserwacje fenotypowe
  31 331 172 x 1 31 998 730 x 1 34 000 684 x 1
- $\boldsymbol{b}$ estymatory efektów stałych
  463 576 x 1 569 385 x 1 600 158 x 1
- $\boldsymbol{a}$ predyktory wartości hodowlanych
  37 354 579 x 1 38 389 025 x 1 40 496 318 x 1
- $\boldsymbol{e}$ predyktory efektów błędu
  31 331 172 x 1 31 998 730 x 1 34 000 684 x 1
- $\boldsymbol{X}$ 34 000 684 x 600 158
- $\boldsymbol{Z}$ 34 000 684 x 40 496 318

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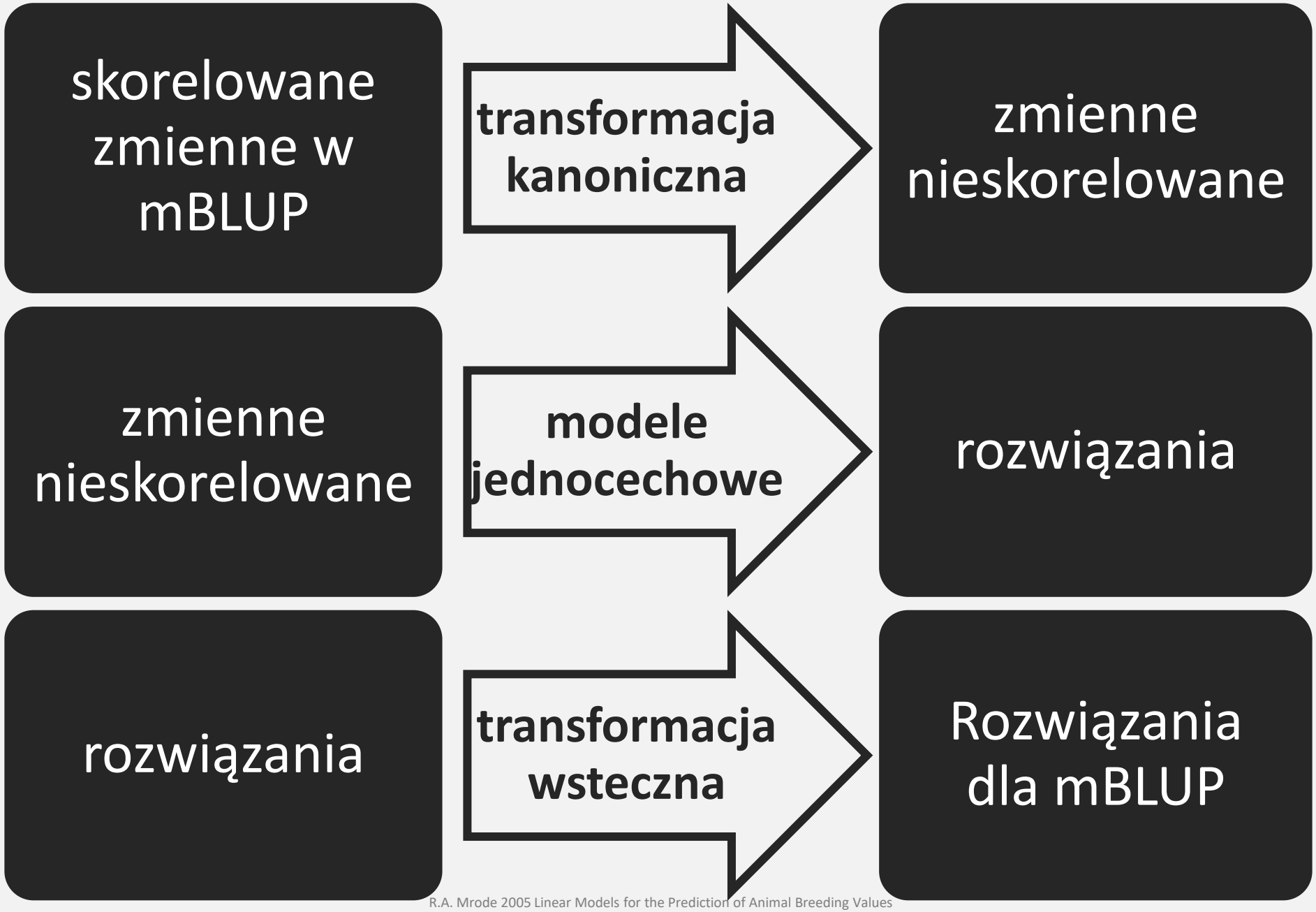
Metody redukcji wymiarów modelu

Canonical transformation

Canonical transformation



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Canonical transformation

- Thompson (1977)
  - Identyczne macierze wystąpień
  - Nie ma brakujących danych
  - Dwa efekty losowe *a* i *e*

- Ducrocq & Besbes (1993)
  - Brakujące rekordy danych

- Misztal (1995)
  - Więcej niż dwa efekty losowe

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Canonical transformation

$$\begin{bmatrix} \boldsymbol{y_1} \\ \boldsymbol{y_2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{X_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{X_2} \end{bmatrix} \begin{bmatrix} \boldsymbol{b_1} \\ \boldsymbol{b_2} \end{bmatrix} + \begin{bmatrix} \boldsymbol{Z_1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Z_2} \end{bmatrix} \begin{bmatrix} \boldsymbol{a_1} \\ \boldsymbol{a_2} \end{bmatrix} + \begin{bmatrix} \boldsymbol{e_1} \\ \boldsymbol{e_2} \end{bmatrix}$$

$$\boldsymbol{G_0} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \qquad \boldsymbol{R_0} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}$$

$$\boldsymbol{var} \begin{bmatrix} \boldsymbol{a_1} \\ \boldsymbol{a_2} \\ \boldsymbol{e_1} \\ \boldsymbol{e_2} \end{bmatrix} = \begin{bmatrix} g_{11}\boldsymbol{A} & g_{12}\boldsymbol{A} & \boldsymbol{0} & \boldsymbol{0} \\ g_{21}\boldsymbol{A} & g_{22}\boldsymbol{A} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & r_{11}\boldsymbol{I} & r_{12}\boldsymbol{I} \\ \boldsymbol{0} & \boldsymbol{0} & r_{21}\boldsymbol{I} & r_{22}\boldsymbol{I} \end{bmatrix}$$

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Canonical transformation

- Wektor skorelowanych obserwacji: $\boldsymbol{y} = \begin{bmatrix} \boldsymbol{y_1} \\ \boldsymbol{y_2} \end{bmatrix}$

$$\boldsymbol{var}(\boldsymbol{y}) \sim \boldsymbol{G_0} + \boldsymbol{R_0}$$

$$\boldsymbol{var}[\boldsymbol{y}] = \begin{bmatrix} g_{11}\boldsymbol{A} & g_{12}\boldsymbol{A} & \boldsymbol{0} & \boldsymbol{0} \\ g_{21}\boldsymbol{A} & g_{22}\boldsymbol{A} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & r_{11}\boldsymbol{I} & r_{12}\boldsymbol{I} \\ \boldsymbol{0} & \boldsymbol{0} & r_{21}\boldsymbol{I} & r_{22}\boldsymbol{I} \end{bmatrix}$$

- Zdefiniować macierz $\boldsymbol{Q}$: $\boldsymbol{Q}\boldsymbol{G_0}\boldsymbol{Q}^{\boldsymbol{T}} = \boldsymbol{W}$ oraz $\boldsymbol{Q}\boldsymbol{R_0}\boldsymbol{Q}^{\boldsymbol{T}} = \boldsymbol{I}$

$\boldsymbol{W}$ → macierz diagonalna

$\boldsymbol{I}$ → macierz jednostkowa

- Wektor nieskorelowanych obserwacji: $\boldsymbol{y}^* = \boldsymbol{Q}\boldsymbol{y}$

$$\boldsymbol{var}(\boldsymbol{y}^*) \sim \boldsymbol{W} + \boldsymbol{I}$$

$$\boldsymbol{var}[\boldsymbol{y}^*] = \begin{bmatrix} w_{11}\boldsymbol{A} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & w_{22}\boldsymbol{A} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & i_{11}\boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & i_{22}\boldsymbol{I} \end{bmatrix}$$

$\boldsymbol{var}(\boldsymbol{y}^*)$ → macierz diagonalna = brak korelacji $\boldsymbol{y}^*$

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Canonical transformation

- Model **jednocechowy** dla transformowanej cechy i

$$\boldsymbol{y_i^*} = \boldsymbol{X}\boldsymbol{b_i^*} + \boldsymbol{Z}\boldsymbol{a_i^*} + \boldsymbol{e_i^*}$$

- MME

$$\begin{bmatrix} \boldsymbol{X}^T\boldsymbol{X} & \boldsymbol{X}^T\boldsymbol{Z} \\ \boldsymbol{Z}^T\boldsymbol{X} & \boldsymbol{Z}^T\boldsymbol{Z} + \boldsymbol{A}^{-\mathbf{1}}\alpha_i \end{bmatrix} \begin{bmatrix} \boldsymbol{b_i^*} \\ \boldsymbol{a_i^*} \end{bmatrix} = \begin{bmatrix} \boldsymbol{X}^T\boldsymbol{y_i^*} \\ \boldsymbol{Z}^T\boldsymbol{y_i^*} \end{bmatrix}$$

$$\alpha_i = \frac{1}{w_{ii}} \qquad \mathbf{W} = \begin{bmatrix} w_{ii} & 0 \\ 0 & w_{ii} \end{bmatrix}$$

$$\boldsymbol{var}\left(\boldsymbol{a_i^*}\right) = \boldsymbol{A}w_{ii} \qquad \boldsymbol{var}\left(\boldsymbol{e_i^*}\right) = \boldsymbol{I} \qquad \boldsymbol{var}\left(\boldsymbol{y_i^*}\right) = \boldsymbol{Z}\boldsymbol{A}\boldsymbol{Z}^{\boldsymbol{T}}w_{ii} + \boldsymbol{I}$$

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Canonical transformation

- Transformacja wsteczna

$$\boldsymbol{b_i} = \boldsymbol{Q}^{-1}\boldsymbol{b_i^*}$$

$$\boldsymbol{a_i} = \boldsymbol{Q}^{-1}\boldsymbol{a_i^*}$$

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Canonical transformation - przykład

Canonical transformation - przykład

Dane

Calf	Sex	Sire	Dam	WWG	PWG
4	Male	1	–	4.5	6.8
5	Female	3	2	2.9	5.0
6	Female	1	2	3.9	6.8
7	Male	4	5	3.5	6.0
8	Male	3	6	5.0	7.5

WWG, pre-weaning gain; PWG, post-weaning gain.

$$\boldsymbol{G_0} = \begin{bmatrix} 20 & 18 \\ 18 & 40 \end{bmatrix} \quad \boldsymbol{R_0} = \begin{bmatrix} 40 & 11 \\ 11 & 30 \end{bmatrix} \quad \boldsymbol{Q} = \begin{bmatrix} 0.17 & -0.08 \\ 0.02 & 0.18 \end{bmatrix}$$

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Canonical transformation - przykład

Kroki wyznaczania Q

1. Obliczyć wartości i wektory własne macierzy $\boldsymbol{R_0}$

www.youtube.com/watch?v=HH8pouRwphA

$$\boldsymbol{R_0} = \boldsymbol{U}\boldsymbol{B}\boldsymbol{U}^{\boldsymbol{T}} \qquad \boldsymbol{R_0} = \begin{bmatrix} 40 & 11 \\ 11 & 30 \end{bmatrix}$$

Wartości własne $\mathrm{diag}(\boldsymbol{B}) = \boldsymbol{B} = \begin{bmatrix} 47.08 & 0 \\ 0 & 22.92 \end{bmatrix}$

Wektory własne $\boldsymbol{U} = \begin{bmatrix} 0.84 & -0.54 \\ 0.54 & 0.84 \end{bmatrix}$

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Canonical transformation - przykład

Kroki wyznaczania Q

$$\boldsymbol{B} = \begin{bmatrix} 47.08 & 0 \\ 0 & 22.92 \end{bmatrix} \quad \boldsymbol{U} = \begin{bmatrix} 0.84 & -0.54 \\ 0.54 & 0.84 \end{bmatrix}$$

2. Wyznaczyć macierz $\boldsymbol{P}$

$$\boldsymbol{P} = \boldsymbol{U}\sqrt{\boldsymbol{B}^{-1}}\boldsymbol{U}^{\boldsymbol{T}} = \begin{bmatrix} 0.16 & -0.03 \\ -0.03 & 0.19 \end{bmatrix}$$

3. Wyznaczyć macierz $\boldsymbol{P}\boldsymbol{G}_{\boldsymbol{0}}\boldsymbol{P}^{\boldsymbol{T}}$

$$\boldsymbol{G}_{\boldsymbol{0}} = \begin{bmatrix} 20 & 18 \\ 18 & 40 \end{bmatrix} \quad \boldsymbol{P}\boldsymbol{G}_{\boldsymbol{0}}\boldsymbol{P}^{\boldsymbol{T}} = \begin{bmatrix} 0.40 & 0.26 \\ 0.26 & 1.27 \end{bmatrix}$$

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Canonical transformation - przykład

Kroki wyznaczania Q

4. Obliczyć wartości i wektory własne macierzy $\boldsymbol{PG_0P^T}$

$$\boldsymbol{PG_0P^T} = \boldsymbol{LWL^T}$$

$$\boldsymbol{W} = \begin{bmatrix} 0.33 & 0 \\ 0 & 1.34 \end{bmatrix} \quad \boldsymbol{L} = \begin{bmatrix} 0.96 & 0.27 \\ -0.27 & 0.96 \end{bmatrix}$$

5. Wyznaczyć macierz $\boldsymbol{Q}$

$$\boldsymbol{P} = \begin{bmatrix} 0.16 & -0.03 \\ -0.03 & 0.19 \end{bmatrix} \quad \boldsymbol{Q} = \boldsymbol{L^TP} = \begin{bmatrix} 0.17 & -0.08 \\ 0.02 & 0.18 \end{bmatrix}$$

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Canonical transformation - przykład

Transformacja cech

Table 5.2. Pre-weaning gain and post-weaning gain for beef calves on the original and transformed scales.

				Original scale		Transformed scale	
Calf	Sex	Sire	Dam	WWG	PWG	VAR1	VAR2
4	Male	1	–	4.5	6.8	0.208	1.269
5	Female	3	2	2.9	5.0	0.085	0.926
6	Female	1	2	3.9	6.8	0.109	1.259
7	Male	4	5	3.5	6.0	0.106	1.112
8	Male	3	6	5.0	7.5	0.236	1.400

VAR1 and VAR2, transformed variables for WWG and PWG, respectively.

Np. dla osobnika 4

$$\boldsymbol{y}_4^* = \boldsymbol{Q}\boldsymbol{y}_4 = \begin{bmatrix} 0.17 & -0.08 \\ 0.02 & 0.18 \end{bmatrix} \begin{bmatrix} 4.5 \\ 6.8 \end{bmatrix} = \begin{bmatrix} 0.208 \\ 1.269 \end{bmatrix}$$

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Canonical transformation - przykład

Rozwiązania MME dla WWG*

$$
\begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \\ \hat{a}_4 \\ \hat{a}_5 \\ \hat{a}_6 \\ \hat{a}_7 \\ \hat{a}_8 \end{bmatrix}
=
\begin{bmatrix}
3.0 & 0.0 & 0.000 & 0.000 & 0.000 & 1.000 & 0.000 & 0.000 & 1.000 & 1.000 \\
0.0 & 2.0 & 0.000 & 0.000 & 0.000 & 0.000 & 1.000 & 1.000 & 0.000 & 0.000 \\
0.0 & 0.0 & 5.584 & 1.523 & 0.000 & -2.031 & 0.000 & -3.046 & 0.000 & 0.000 \\
0.0 & 0.0 & 1.523 & 6.092 & 1.523 & 0.000 & -3.046 & -3.046 & 0.000 & 0.000 \\
0.0 & 0.0 & 0.000 & 1.523 & 6.092 & 0.000 & -3.046 & 1.523 & 0.000 & -3.046 \\
1.0 & 0.0 & -2.031 & 0.000 & 0.000 & 6.584 & 1.523 & 0.000 & -3.046 & 0.000 \\
0.0 & 1.0 & 0.000 & -3.046 & -3.046 & 1.523 & 8.615 & 0.000 & -3.046 & 0.000 \\
0.0 & 1.0 & -3.046 & -3.046 & 1.523 & 0.000 & 0.000 & 8.615 & 0.000 & -3.046 \\
1.0 & 0.0 & 0.000 & 0.000 & 0.000 & -3.046 & -3.046 & 0.000 & 7.092 & 0.000 \\
1.0 & 0.0 & 0.000 & 0.000 & -3.046 & 0.000 & 0.000 & -3.046 & 0.000 & 7.092
\end{bmatrix}^{-1}
\begin{bmatrix} 0.549 \\ 0.194 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.208 \\ 0.085 \\ 0.108 \\ 0.105 \\ 0.235 \end{bmatrix}
$$

estymatory dla cech transformowanych

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Canonical transformation - przykład

Rozwiązania: mBLUB

Effects	Multivariate analysis traits WWG	PWG
*Sex**		
1	4.361	6.800
2	3.397	5.880
Animal		
1	0.151	0.280
2	−0.015	−0.008
3	−0.078	−0.170
4	−0.010	−0.013
5	−0.270	−0.478
6	0.276	0.517
7	−0.316	−0.479
8	0.244	0.392

Rozwiązania – transformacja kanoniczna

	Canonical scale		Original scale	
Effects	VAR1	VAR2	WWG	PWG
Sex				
1	0.185	1.266	4.361	6.800
2	0.098	1.089	3.397	5.880
Animal				
1	0.003	0.052	0.151	0.280
2	−0.002	−0.002	−0.015	−0.008
3	0.000	−0.031	−0.078	−0.170
4	−0.001	−0.002	−0.010	−0.013
5	−0.007	−0.088	−0.270	−0.478
6	0.005	0.095	0.276	0.517
7	−0.015	−0.089	−0.316	−0.479
8	0.009	0.073	0.244	0.392

Np. dla osobnika 1

$$\boldsymbol{a_1} = \boldsymbol{Q}^{-1}\boldsymbol{a_1^*} = \begin{bmatrix} 5.77 & 2.60 \\ -0.55 & 5.45 \end{bmatrix} \begin{bmatrix} 0.003 \\ 0.052 \end{bmatrix} = \begin{bmatrix} 0.151 \\ 0.280 \end{bmatrix}$$

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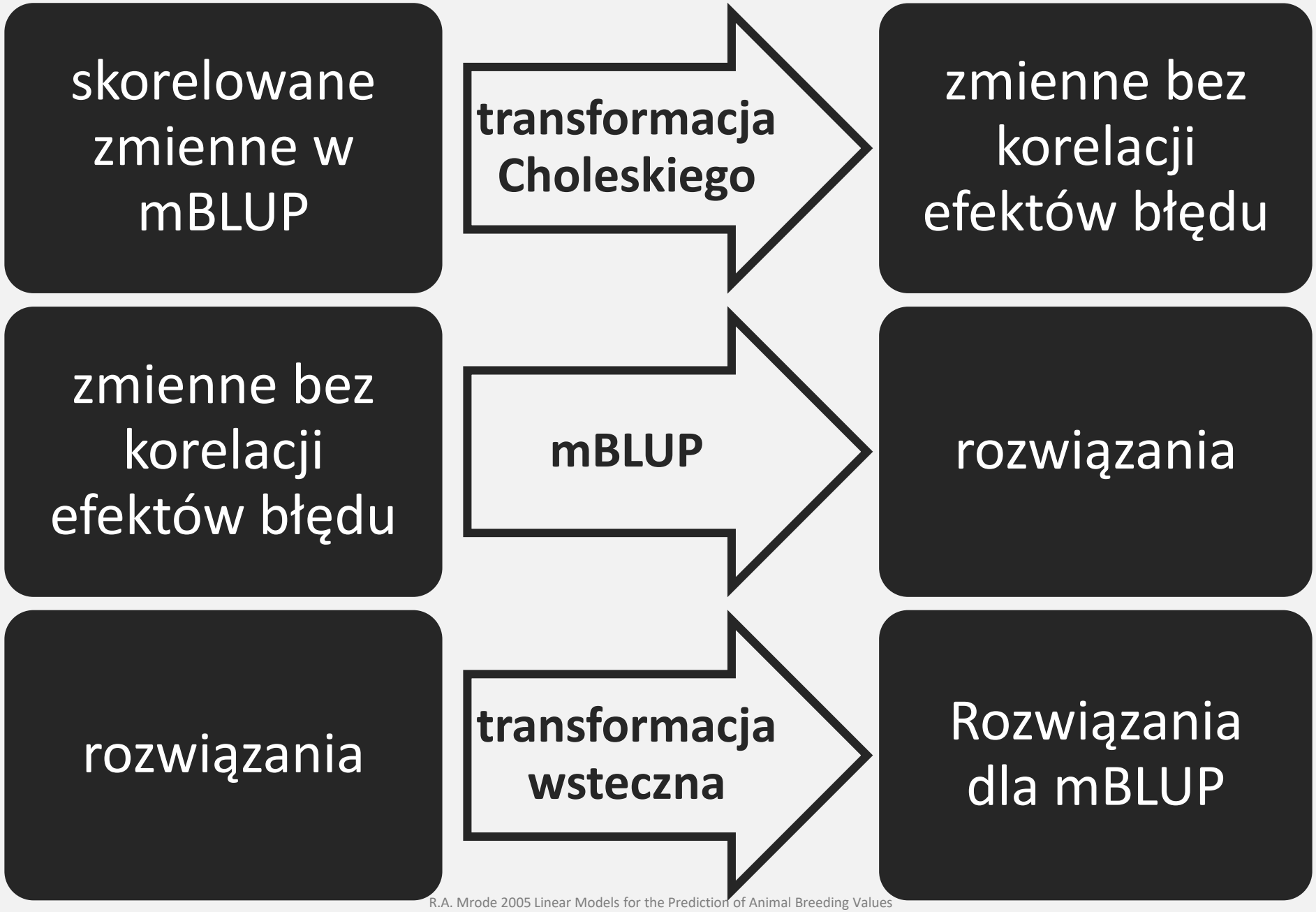
Cholesky transformation

Cholesky transformation



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Cholesky transformation

- Quaas (1984)
- Brakujące wartości cech

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Cholesky transformation

- Wektor skorelowanych obserwacji: $\boldsymbol{y} = \begin{bmatrix} \boldsymbol{y_1} \\ \boldsymbol{y_2} \end{bmatrix}$ $\quad \boldsymbol{var}[\boldsymbol{y}] = \begin{bmatrix} g_{11}\boldsymbol{A} & g_{12}\boldsymbol{A} & \boldsymbol{0} & \boldsymbol{0} \\ g_{21}\boldsymbol{A} & g_{22}\boldsymbol{A} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & r_{11}\boldsymbol{I} & r_{12}\boldsymbol{I} \\ \boldsymbol{0} & \boldsymbol{0} & r_{21}\boldsymbol{I} & r_{22}\boldsymbol{I} \end{bmatrix}$

$$\boldsymbol{var}(\boldsymbol{y}) \sim \boldsymbol{G_0} + \boldsymbol{R_0}$$

- Przeprowadzić dekompozycję Choleskiego i zdefiniować macierz $\boldsymbol{T}$:
  $\boldsymbol{R_0} = \boldsymbol{T}\boldsymbol{T}^{\boldsymbol{T}}$
  $\boldsymbol{T}$: lower triangular $\quad \boldsymbol{T}^{\boldsymbol{T}}$: upper triangular

- Wektor obserwacji o nieskorelowanych błędach: $\boldsymbol{y}^* = \boldsymbol{T}^{-\boldsymbol{1}}\boldsymbol{y}$

$$\boldsymbol{var}(\boldsymbol{y}^*) \sim \boldsymbol{T}^{-\boldsymbol{1}}\boldsymbol{G_0}\boldsymbol{T}^{-\boldsymbol{1}\boldsymbol{T}} + \boldsymbol{I} = \boldsymbol{M} + \boldsymbol{I}$$

$$\boldsymbol{var}[\boldsymbol{y}] = \begin{bmatrix} m_{11}\boldsymbol{A} & m_{12}\boldsymbol{A} & \boldsymbol{0} & \boldsymbol{0} \\ m_{21}\boldsymbol{A} & m_{22}\boldsymbol{A} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & r_{11}\boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & r_{22}\boldsymbol{I} \end{bmatrix}$$

w przypadku brakujących cech $y_{ij}^* = 0$

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Cholesky transformation

- Model wielocechowy dla cech transformowanych

$$\begin{bmatrix} \boldsymbol{y}_1^* \\ \boldsymbol{y}_2^* \end{bmatrix} = \begin{bmatrix} \boldsymbol{X}_1 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{X}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{b}_1^* \\ \boldsymbol{b}_2^* \end{bmatrix} + \begin{bmatrix} \boldsymbol{Z}_1 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Z}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{a}_1^* \\ \boldsymbol{a}_2^* \end{bmatrix} + \begin{bmatrix} \boldsymbol{e}_1^* \\ \boldsymbol{e}_2^* \end{bmatrix}$$

$$\boldsymbol{var}(\boldsymbol{y}^*) \sim \boldsymbol{M} + \boldsymbol{I}$$

$$\mathbf{M} = \begin{bmatrix} m_{ii} & m_{ij} \\ m_{ij} & m_{ii} \end{bmatrix} \quad \mathbf{M}^{-1} = \begin{bmatrix} m^{ii} & m^{ij} \\ m^{ij} & m^{ii} \end{bmatrix}$$

- MME

$$\begin{bmatrix} \boldsymbol{X}_1^T\boldsymbol{X}_1 & 0 & \boldsymbol{X}_1^T\boldsymbol{Z}_1 & 0 \\ 0 & \boldsymbol{X}_2^T\boldsymbol{X}_2 & 0 & \boldsymbol{X}_2^T\boldsymbol{Z}_2 \\ \boldsymbol{Z}_1^T\boldsymbol{X}_1 & 0 & \boldsymbol{Z}_1^T\boldsymbol{Z}_1 + \boldsymbol{A}^{-1}m^{11} & \boldsymbol{A}^{-1}m^{12} \\ 0 & \boldsymbol{Z}_2^T\boldsymbol{X}_2 & \boldsymbol{A}^{-1}m^{21} & \boldsymbol{Z}_2^T\boldsymbol{Z}_2 + \boldsymbol{A}^{-1}m^{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{b}_1^* \\ \boldsymbol{b}_2^* \\ \boldsymbol{a}_1^* \\ \boldsymbol{a}_1^* \end{bmatrix} = \begin{bmatrix} \boldsymbol{X}_1^T\boldsymbol{y}_1^* \\ \boldsymbol{X}_2^T\boldsymbol{y}_2^* \\ \boldsymbol{Z}_1^T\boldsymbol{y}_1^* \\ \boldsymbol{Z}_2^T\boldsymbol{y}_2^* \end{bmatrix}$$

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Cholesky transformation

- Standardowy model wielocechowy

$$\begin{bmatrix} \boldsymbol{X}_1^T \boldsymbol{R}_{11} \boldsymbol{X}_1 & \boldsymbol{X}_1^T \boldsymbol{R}_{12} \boldsymbol{X}_2 & \boldsymbol{X}_1^T \boldsymbol{R}_{11} \boldsymbol{Z}_1 & \boldsymbol{X}_1^T \boldsymbol{R}_{12} \boldsymbol{Z}_2 \\ \boldsymbol{X}_2^T \boldsymbol{R}_{21} \boldsymbol{X}_1 & \boldsymbol{X}_2^T \boldsymbol{R}_{22} \boldsymbol{X}_2 & \boldsymbol{X}_2^T \boldsymbol{R}_{21} \boldsymbol{Z}_1 & \boldsymbol{X}_2^T \boldsymbol{R}_{22} \boldsymbol{Z}_2 \\ \boldsymbol{Z}_1^T \boldsymbol{R}_{11} \boldsymbol{X}_1 & \boldsymbol{Z}_1^T \boldsymbol{R}_{12} \boldsymbol{X}_2 & \boldsymbol{Z}_1^T \boldsymbol{R}_{22} \boldsymbol{Z}_1 + \boldsymbol{A}^{-1} g^{11} & \boldsymbol{Z}_1^T \boldsymbol{R}_{12} \boldsymbol{Z}_2 + \boldsymbol{A}^{-1} g^{12} \\ \boldsymbol{Z}_2^T \boldsymbol{R}_{21} \boldsymbol{X}_1 & \boldsymbol{Z}_2^T \boldsymbol{R}_{22} \boldsymbol{X}_2 & \boldsymbol{Z}_2^T \boldsymbol{R}_{22} \boldsymbol{Z}_1 + \boldsymbol{A}^{-1} g^{21} & \boldsymbol{Z}_2^T \boldsymbol{R}_{22} \boldsymbol{Z}_2 + \boldsymbol{A}^{-1} g^{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \\ \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{X}_1^T \boldsymbol{y}_1 \\ \boldsymbol{X}_2^T \boldsymbol{y}_2 \\ \boldsymbol{Z}_1^T \boldsymbol{y}_1 \\ \boldsymbol{Z}_2^T \boldsymbol{y}_2 \end{bmatrix}$$

- Model wielocechowy dla cech transformowanych

$$\begin{bmatrix} \boldsymbol{X}_1^T \boldsymbol{X}_1 & 0 & \boldsymbol{X}_1^T \boldsymbol{Z}_1 & 0 \\ 0 & \boldsymbol{X}_2^T \boldsymbol{X}_2 & 0 & \boldsymbol{X}_2^T \boldsymbol{Z}_2 \\ \boldsymbol{Z}_1^T \boldsymbol{X}_1 & 0 & \boldsymbol{Z}_1^T \boldsymbol{Z}_1 + \boldsymbol{A}^{-1} m^{11} & \boldsymbol{A}^{-1} m^{12} \\ 0 & \boldsymbol{Z}_2^T \boldsymbol{X}_2 & \boldsymbol{A}^{-1} m^{21} & \boldsymbol{Z}_2^T \boldsymbol{Z}_2 + \boldsymbol{A}^{-1} m^{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{b}_1^* \\ \boldsymbol{b}_2^* \\ \boldsymbol{a}_1^* \\ \boldsymbol{a}_1^* \end{bmatrix} = \begin{bmatrix} \boldsymbol{X}_1^T \boldsymbol{y}_1^* \\ \boldsymbol{X}_2^T \boldsymbol{y}_2^* \\ \boldsymbol{Z}_1^T \boldsymbol{y}_1^* \\ \boldsymbol{Z}_2^T \boldsymbol{y}_2^* \end{bmatrix}$$

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Cholesky transformation

- Transformacja wsteczna

$$\boldsymbol{b_i} = \boldsymbol{T}\boldsymbol{b_i^*}$$

$$\boldsymbol{a_i} = \boldsymbol{T}\boldsymbol{a_i^*}$$

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Cholesky transformation - przykład

Cholesky transformation - przykład

Dane

Calf	Sex	Sire	Dam	WWG (kg)	PWG (kg)
4	Male	1	–	4.5	–
5	Female	3	2	2.9	5.0
6	Female	1	2	3.9	6.8
7	Male	4	5	3.5	6.0
8	Male	3	6	5.0	7.5
9	Female	7	8	4.0	–

$$R_m = \mathbf{40} \quad \boldsymbol{R}_o = \begin{bmatrix} 40 & 11 \\ 11 & 30 \end{bmatrix} \quad \boldsymbol{G_0} = \begin{bmatrix} 20 & 18 \\ 18 & 40 \end{bmatrix} \quad \boldsymbol{T} = \begin{bmatrix} 6.3 & 0 \\ 1.7 & 5.2 \end{bmatrix}$$

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Cholesky transformation - przykład

Wyznaczanie T → Cholesky decomposition

www.youtube.com/watch?v=TprfUB3nI8Y

$$\boldsymbol{R_0} = \boldsymbol{TT^T}$$

- Elementy diagonalne: $t_{ii} = \sqrt{r_{ii} - \sum_{j=1}^{i-1} t_{ij}^2}$
- Elementy dolnej poza-przekątnej:

$$t_{ik} = \frac{1}{t_{kk}} \left(r_{ik} - \sum_{j=1}^{k-1} t_{ij}\ t_{kj} \right)$$

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Cholesky transformation - przykład

Transformacja cech

				Original traits		Transformed traits	
Calf	Sex	Sire	Dam	WWG	PWG	y_1^*	y_2^*
4	Male	1	–	4.5	–	0.712	–
5	Female	3	2	2.9	5.0	0.456	0.809
6	Female	1	2	3.9	6.8	0.617	1.103
7	Male	4	5	3.5	6.0	0.553	0.970
8	Male	3	6	5.0	7.5	0.791	1.179
9	Female	7	8	4.0	–	0.632	–

Np. dla osobnika 4, cecha i=1

$$y_{14}^* = t^{11} y_{14}$$

Np. dla osobnika 5, cechy i=1 oraz j=2

$$y_{15}^* = t^{11} y_{15}$$

$$y_{25}^* = t^{22} y_{25} + t^{12} y_{15}$$

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Cholesky transformation - przykład

Rozwiązania: mBLUP

	Multivariate analysis	
Effects	WWG	PWG
Sex		
1	4.327	6.793
2	3.598	5.966
Animal		
1	0.154	0.288
2	−0.059	−0.053
3	−0.061	−0.163
4	0.027	0.036
5	−0.307	−0.521
6	0.235	0.477
7	−0.280	−0.452
8	0.272	0.407
9	0.077	0.051

Rozwiązania – transformacja Choleskiego

	Transformed scale		Original scale	
Effects	WWG	PWG	WWG	PWG
Sex				
1	0.684	1.079	4.327	6.793
2	0.569	0.958	3.598	5.966
Animal				
1	0.024	0.047	0.154	0.288
2	−0.009	−0.007	−0.059	−0.053
3	−0.010	−0.028	−0.061	−0.163
4	0.004	0.005	0.027	0.036
5	−0.048	−0.084	−0.307	−0.521
6	0.037	0.079	0.235	0.477
7	−0.044	−0.072	−0.280	−0.452
8	0.043	0.064	0.272	0.407
9	0.012	0.006	0.077	0.051

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1. Transformacja kanoniczna
2. Transformacja Choleskiego

1. Kiedy stosujemy transformację kanoniczną?
2. Co nam daje tranzformacja kanoniczna
3. Co nam daje transformacja Choleskiego?